Signed Cryptographic Program Verification with Typed CryptoLine

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Outline

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Practical Cryptography

- Cryptographic program is written in C or ASM for efficiency.
- Computation over large finite field is not trivial in C and ASM.
- Split a large number into several smaller numbers (a.k.a. limbs).
  (e.g. 4 or 5 uint64_t/register to store 255-bit keys for Curve25519)
- Computation over limbs is error-prone.
- A simple bug can cause catastrophic damages.
  (e.g. a missing bound check in Heartbleed)

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In this work, we focus on implementation written in C.

Functional Correctness

So.... How to achieve the functional correctness?

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Verification

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  - many human-added annotations.

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    - SMT solver cannot deal with large integers multiplication well
- DSL + SMT Solver + Algebra Solver (PTW+18)
  - Untyped CRYPTOLINE (only unsigned)
  - Target: ASM (some real-word examples in OpenSSL)
  - integer size is fixed (32/64 bit register)

SMT: Satisfiability modulo theories
DSL: Domain-specific language

Goal

- More real-world examples.
- Try to verify the C implementation once instead of ASM for every platforms.
  - most implementation now are still written in C instead of human-optimized ASM
- Less verification effort and friendly to normal cryptographic library developers.

Target Cryptographic Libraries

- OpenSSL: **UBIQUITOUS**
- BoringSSL: Chrome, Android
- NaCl: reference implementation
- wolfSSL: embedded systems
- Bitcoin's libsecp256k1: ECDSA used by **MANY** cryptocurrencies (Ethereum, Zcash, Ripple, $\cdots$)

What Curves We Verified

- OpenSSL: — 32/64: integer size
  - NIST P-224 : $2^{224} - 2^{96} + 1$ (unsigned 64)
  - NIST P-256 : $2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$ (unsigned 64)
  - NIST P-521 : $2^{521} - 1$ (unsigned 64)
  - Curve25519 : $2^{255} - 19$ (unsigned 64, signed 32)
- BoringSSL: Curve25519 (unsigned 64)
- NaCl: Curve25519 (unsigned 64, signed 64)
- wolfSSL: Curve25519 (same as OpenSSL's) (signed 32)
- Bitcoin: Secp256k1 ($2^{256} - 2^{32} - 2^9 - 2^8 - 2^7 - 2^6 - 2^4 - 1$)
  (unsigned, signed)

Contribution

- **Typed** CRYPTOLINE – unsigned and signed, arbitrary size integers
  - type system (type checking & type inference)
- A GCC plugin that translates GIMPLECRYPTOLINE into **Typed** CRYPTOLINE
- GIMPLECRYPTOLINE – a subset of GIMPLE
  - GIMPLE: a GCC IR used in machine-independent optimization
- Verify GIMPLE code after machine-independent optimization
- First to verify signed C implementation in cryptographic libraries used in industry
- Found a bug in NaCl's Curve25519 - Case study

Outline

Typed CryptoLine Program

- Program - instructions
- Specification
  - Assumption (Precondition)
  - Assertion (Postcondition)
  - Properties {algebra && range}
    - range: variables should be in a proper range (e.g. $a < 2^{51}$)
      checked by SMT solver (Boolector, MathSAT, Z3 $\cdots$)
    - algebra: mathematical properties (e.g. $c = a \times b$)
      checked by algebraic solver (Sage, Singular, Mathematica $\cdots$)
- Hoare triple: {assumption} program {assertion}

Typed CryptoLine Program Example - Naive Addition

```
proc main (uint64 a0, uint64 a1, uint64 b0, uint64 b1) =
{
  true // algebraic prop; true means no assumption
  &&
  and [  // range prop
    a0 <u (2**63)@64, a1 <u (2**63)@64,
    b0 <u (2**63)@64, b1 <u (2**63)@64
  ]
}
add c0 a0 b0; // c0 = a0 + b0
add c1 a1 b1; // c1 = a1 + b1
{
  limbs 64 [c0, c1]
  =
  limbs 64 [a0, a1] + limbs 64 [b0, b1]
  &&
  and [
    c0 >=u a0, c1 >=u a1 // true iff not overflow
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$$\text{limbs } 64\ [a_0, a_1, \cdots, a_n] = \sum_{i=0}^{n} a_i \times 2^{64 \times i}$$

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$$2^{63} - 1 + 2^{63} - 1 = 2^{64} - 2 \leq 2^{64} - 1$$

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}
```

$$2^{63} + 2^{63} = 2^{64} \not\leq 2^{64} - 1$$
$$2^{64} = 0 \; (mod \; 2^{64})$$

Program Safety Check by SMT Solver

Safety in our context means that following kinds of errors do not exist.

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- Cast between types
  (uint64 $\leftrightarrow$ int64, uint64 $\leftrightarrow$ uint32)
  Value preserving casting (vpc)

2's complement representation for signed integers

uint4 $\leftrightarrow$ int4

$(0111)_2$ =	7	(unsigned) =	7	(signed)
$(1111)_2$ =	15	(unsigned) =	-1	(signed)

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BUG (vpc) or on purpose (cast)

Counterexample by SMT solver

Typed CRYPTOLINE Program Example - Cast v.s. VPC

```
proc main (uint64 a ,uint64 b)=
{
 true
 &&
 and [
     a <u (2**63), b <u (2**63)
  ]
}
cast wa@int64 a;
cast wb@int64 b;
mul c wa wb;
{ ... }
```

✓

```
proc main (uint64 a ,uint64 b)=
{
 true
 &&
 and [
     a <u (2**63), b <u (2**63)
  ]
}
vpc wa@int64 a;
vpc wb@int64 b;
mul c wa wb;
{ ... }
```

✓

Figure: cast = vpc in some cases

under the assumption, sign bit will never be 1.

Typed CRYPTOLINE Program Example - VPC Error

```
proc main (uint64 a ,uint64 b)=
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✗

Figure: cast $\neq$ vpc in some cases

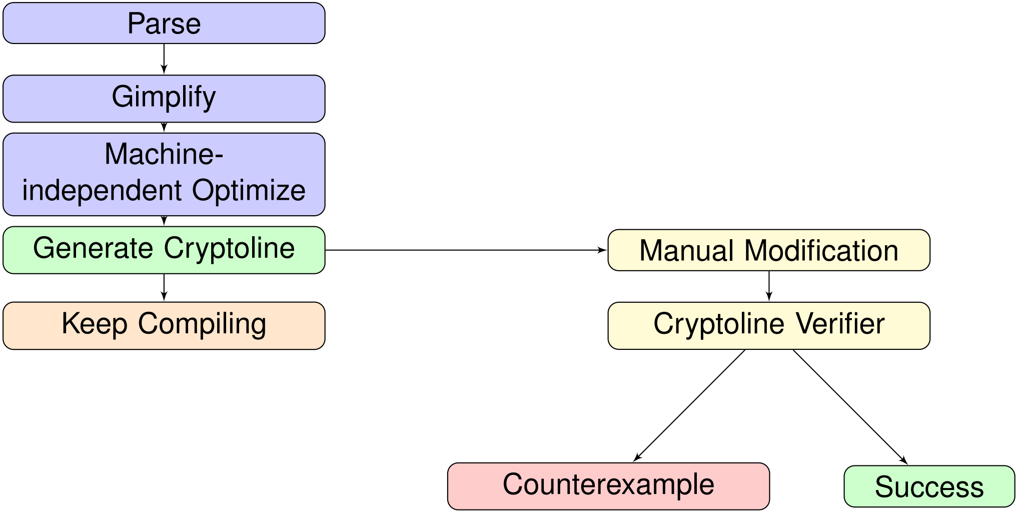
$$2^{63} = (100\ldots0)_2$$

Outline

GCC Plugin

- Introduced in GCC 4.5.0
- Let us add custom optimization passes
- Able to access AST (abstract syntax tree)
    - No need to write parser by yourself!

Verification Workflow Using GCC Plugin



GIMPLE Example

```
f0_3 = *f_2(D);
f1_4 = MEM[(const int32_t*)f_2(D)+4B];
    ...
g0_14 = *g_13(D);
g1_15 = MEM[(const int32_t*)g_13(D)+4B];
    ...
h0_24 = f0_3 - g0_14;
h1_25 = f1_4 - g1_15;
    ...
*h_34(D) = h0_24;
MEM[(int32_t*)h_34(D)+4B] = h1_25;
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?LHS = MEM[?RHS] $\Rightarrow$ Load from RHS to LHS

MEM[?LHS] = ?RHS $\Rightarrow$ Store RHS to LHS

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GimpleCryptoLine Example

generated by the plugin automatically. later manually add assumption / assertion.

```
proc main () =
{ true && true }
mov f03 f2_0; // f0_3 = *f_2
mov f14 f2_4; // f1_4 = MEM[(...) f_2 + 4]
        ...
mov g014 g13_0;
mov g115 g13_4;
        ...
sub h024 f03 g014; // h0_24 = f0_3 - g0_14
sub h125 f14 g115;
        ...
mov h34_0 h024; // *h_34 = h0_24
mov h34_4 h125; // MEM[(...) h_34 + 4]
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```

use pointer name and offset to represent the variable

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Outline

Case Study - Field Subtraction in NaCl Curve25519

```c
typedef uint64_t felem;
/* Find the difference of two numbers: output = in - output
 * (note the order of the arguments!)
 */
static void fdifference_backwards(felem *ioutput, const felem *iin) {
  static const int64_t twotothe51 = (1l << 51);
  const int64_t *in = (const int64_t *) iin;
  int64_t *out = (int64_t *) ioutput;

  out[0] = in[0] - out[0]; out[1] = in[1] - out[1];
  out[2] = in[2] - out[2]; out[3] = in[3] - out[3];
  out[4] = in[4] - out[4];

  NEGCHAIN(0, 1); NEGCHAIN(1, 2);
  NEGCHAIN(2, 3); NEGCHAIN(3, 4);
  NEGCHAIN19(4, 0);
  NEGCHAIN(0, 1); NEGCHAIN(1, 2);
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5 uint64 limbs and use signed computation

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  out[0] = in[0] - out[0]; out[1] = in[1] - out[1];
  out[2] = in[2] - out[2]; out[3] = in[3] - out[3];
  out[4] = in[4] - out[4];

  NEGCHAIN(0, 1); NEGCHAIN(1, 2);
  NEGCHAIN(2, 3); NEGCHAIN(3, 4);
  NEGCHAIN19(4, 0);
  NEGCHAIN(0, 1); NEGCHAIN(1, 2);
  NEGCHAIN(2, 3); NEGCHAIN(3, 4);
```

5 uint64 limbs and use signed computation

Case Study - Field Subtraction in NaCl Curve25519

```
typedef uint64_t felem;
/* Find the difference of two numbers: output = in - output
 * (note the order of the arguments!)
 */
static void fdifference_backwards(felem *ioutput, const felem *iin) {
  static const int64_t twotothe51 = (1l << 51);
  const int64_t *in = (const int64_t *) iin;
  int64_t *out = (int64_t *) ioutput;

  out[0] = in[0] - out[0]; out[1] = in[1] - out[1];
  out[2] = in[2] - out[2]; out[3] = in[3] - out[3];
  out[4] = in[4] - out[4];

  NEGCHAIN(0, 1); NEGCHAIN(1, 2);
  NEGCHAIN(2, 3); NEGCHAIN(3, 4);
  NEGCHAIN19(4, 0);
  NEGCHAIN(0, 1); NEGCHAIN(1, 2);
  NEGCHAIN(2, 3); NEGCHAIN(3, 4);
```

5 uint64 limbs and use signed computation

Case Study - Field Subtraction in NaCl Curve25519

```c
typedef uint64_t felem;
/* Find the difference of two numbers: output = in - output
 * (note the order of the arguments!)
 */
static void fdifference_backwards(felem *ioutput, const felem *iin) {
  static const int64_t twotothe51 = (1l << 51);
  const int64_t *in = (const int64_t *) iin;
  int64_t *out = (int64_t *) ioutput;

  out[0] = in[0] - out[0]; out[1] = in[1] - out[1];
  out[2] = in[2] - out[2]; out[3] = in[3] - out[3];
  out[4] = in[4] - out[4];

  NEGCHAIN(0, 1); NEGCHAIN(1, 2);
  NEGCHAIN(2, 3); NEGCHAIN(3, 4);
  NEGCHAIN19(4, 0);
  NEGCHAIN(0, 1); NEGCHAIN(1, 2);
  NEGCHAIN(2, 3); NEGCHAIN(3, 4);
```

5 uint64 limbs and use signed computation

Case Study - Field Subtraction in NaCl Curve25519

```
typedef uint64_t felem;
/* Find the difference of two numbers: output = in - output
 * (note the order of the arguments!)
 */
static void fdifference_backwards(felem *ioutput, const felem *iin) {
  static const int64_t twotothe51 = (1l << 51);
  const int64_t *in = (const int64_t *) iin;
  int64_t *out = (int64_t *) ioutput;

  out[0] = in[0] - out[0]; out[1] = in[1] - out[1];
  out[2] = in[2] - out[2]; out[3] = in[3] - out[3];
  out[4] = in[4] - out[4];

  NEGCHAIN(0, 1); NEGCHAIN(1, 2);
  NEGCHAIN(2, 3); NEGCHAIN(3, 4);
  NEGCHAIN19(4, 0);
  NEGCHAIN(0, 1); NEGCHAIN(1, 2);
  NEGCHAIN(2, 3); NEGCHAIN(3, 4);
```

5 uint64 limbs and use signed computation

```c
int64_t t;

#define NEGCHAIN(a,b) \
  t = out[a] >> 63; \
  out[a] += twotothe51 & t; \
  out[b] -= 1 & t;

#define NEGCHAIN19(a,b) \
  t = out[a] >> 63; \
  out[a] += twotothe51 & t; \
  out[b] -= 19 & t;
```

```c
  NEGCHAIN(0, 1);
  NEGCHAIN(1, 2);
  NEGCHAIN(2, 3);
  NEGCHAIN(3, 4);
  NEGCHAIN19(4, 0);
  NEGCHAIN(0, 1);
  NEGCHAIN(1, 2);
  NEGCHAIN(2, 3);
  NEGCHAIN(3, 4);
}
```

Figure: Bitwise tricks (signed right shift) & Reduction chain

```c
int64_t t;

#define NEGCHAIN(a,b) \
  t = out[a] >> 63; \
  out[a] += twotothe51 & t; \
  out[b] -= 1 & t;

#define NEGCHAIN19(a,b) \
  t = out[a] >> 63; \
  out[a] += twotothe51 & t; \
  out[b] -= 19 & t;
```

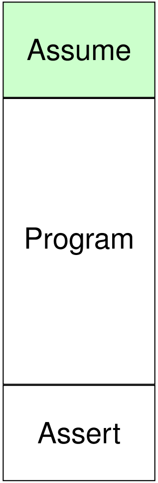
```c
  NEGCHAIN(0, 1);
  NEGCHAIN(1, 2);
  NEGCHAIN(2, 3);
  NEGCHAIN(3, 4);
  NEGCHAIN19(4, 0);
  NEGCHAIN(0, 1);
  NEGCHAIN(1, 2);
  NEGCHAIN(2, 3);
  NEGCHAIN(3, 4);
}
```

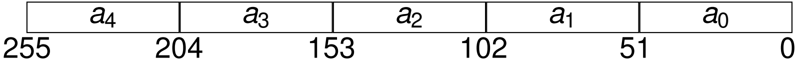
Figure: Bitwise tricks (signed right shift) & Reduction chain

sign_bit(out[a]) == 1/0 $\leftrightarrow$ t is all 1/0

Verify by CRYPTOLINE - Pre-condition

```
proc main (uint64 a0, uint64 a1, uint64 a2, uint64 a3, uint64 a4,
           uint64 b0, uint64 b1, uint64 b2, uint64 b3, uint64 b4) =
{
  true
  &&
  and [
    a0 <u (2**51)@64,
    a1 <u (2**51)@64,
    a2 <u (2**51)@64,
    a3 <u (2**51)@64,
    a4 <u (2**51)@64,
    b0 <u (2**51)@64,
    b1 <u (2**51)@64,
    b2 <u (2**51)@64,
    b3 <u (2**51)@64,
    b4 <u (2**51)@64
  ]
}
```





Verify by CryptoLine - Pre-condition

```
proc main (uint64 a0, uint64 a1, uint64 a2, uint64 a3, uint64 a4,
           uint64 b0, uint64 b1, uint64 b2, uint64 b3, uint64 b4) =
{
  true
  &&
  and [
    a0 <u (2**51)@64,
    a1 <u (2**51)@64,
    a2 <u (2**51)@64,
    a3 <u (2**51)@64,
    a4 <u (2**51)@64,
    b0 <u (2**51)@64,
    b1 <u (2**51)@64,
    b2 <u (2**51)@64,
    b3 <u (2**51)@64,
    b4 <u (2**51)@64
  ]
}
```

Assume
Program
Assert

a_4	a_3	a_2	a_1	a_0

255 204 153 102 51 0

Verify by CRYPTOLINE - Pre-condition

```
proc main (uint64 a0, uint64 a1, uint64 a2, uint64 a3, uint64 a4,
           uint64 b0, uint64 b1, uint64 b2, uint64 b3, uint64 b4) =
{
  true
  &&
  and [
    a0 <u (2**51)@64,
    a1 <u (2**51)@64,
    a2 <u (2**51)@64,
    a3 <u (2**51)@64,
    a4 <u (2**51)@64,
    b0 <u (2**51)@64,
    b1 <u (2**51)@64,
    b2 <u (2**51)@64,
    b3 <u (2**51)@64,
    b4 <u (2**51)@64
  ]
}
```

Assume
Program
Assert

a_4	a_3	a_2	a_1	a_0

255 204 153 102 51 0

Verify by CRYPTOLINE - Pre-condition

```
proc main (uint64 a0, uint64 a1, uint64 a2, uint64 a3, uint64 a4,
           uint64 b0, uint64 b1, uint64 b2, uint64 b3, uint64 b4) =
{
  true
  &&
  and [
    a0 <u (2**51)@64,
    a1 <u (2**51)@64,
    a2 <u (2**51)@64,
    a3 <u (2**51)@64,
    a4 <u (2**51)@64,
    b0 <u (2**51)@64,
    b1 <u (2**51)@64,
    b2 <u (2**51)@64,
    b3 <u (2**51)@64,
    b4 <u (2**51)@64
  ]
}
```

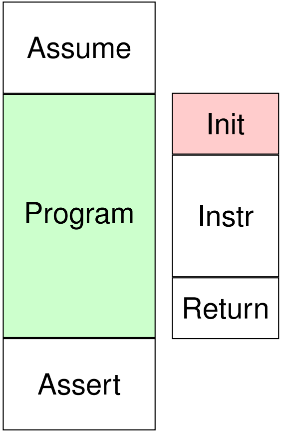
Assume
Program
Assert

a_4	a_3	a_2	a_1	a_0

255 204 153 102 51 0

Verify by CryptoLine - Init type casting

```
vpc iin52_0@int64 a0;
vpc iin52_8@int64 a1;
vpc iin52_16@int64 a2;
vpc iin52_24@int64 a3;
vpc iin52_32@int64 a4;
vpc ioutput53_0@int64 b0;
vpc ioutput53_8@int64 b1;
vpc ioutput53_16@int64 b2;
vpc ioutput53_24@int64 b3;
vpc ioutput53_32@int64 b4;
```



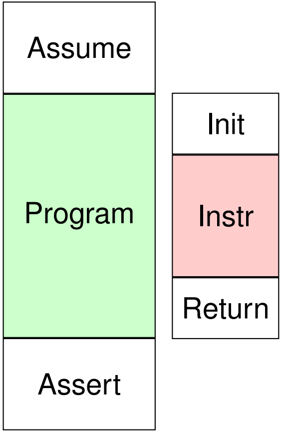
uint64 $\rightarrow$ int64

bridge assumption and program

vpc: value preserve casting (will do safety check)

Verify by CRYPTOLINE - Instructions

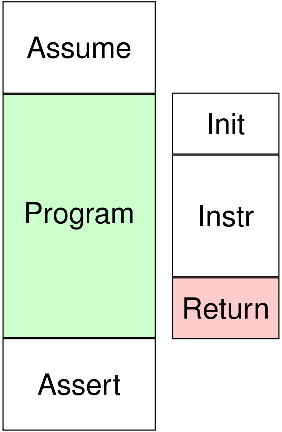
```
(* _1 = MEM[(const int64_t * )iin_52(D)]; *)
mov v1 iin52_0;
(* _2 = MEM[(int64_t * )ioutput_53(D)]; *)
mov v2 ioutput53_0;
(* _3 = _1 - _2; *)
ssub v3 v1 v2;
(* MEM[(int64_t * )ioutput_53(D)] = _3; *)
mov ioutput53_0 v3;
(* _4 = MEM[(const int64_t * )iin_52(D) + 8B]; *)
mov v4 iin52_8;
(* _5 = MEM[(int64_t * )ioutput_53(D) + 8B]; *)
mov v5 ioutput53_8;
(* _6 = _4 - _5; *)
ssub v6 v4 v5;
```



ssub: signed subtraction (usub/ssub explicitly ⇒ type checking, sub ⇒ type inference)

Verify by CryptoLine - Return type casting

```
vpc c0@uint64 ioutput53_0@int64;
vpc c1@uint64 ioutput53_8@int64;
vpc c2@uint64 ioutput53_16@int64;
vpc c3@uint64 ioutput53_24@int64;
vpc c4@uint64 ioutput53_32@int64;
```



int64 $\rightarrow$ uint64

bridge program and assertion

vpc: value preserve casting (will do safety check)

Verify by CRYPTOLINE - Post-condition

```
{
  (limbs 51 [c0, c1, c2, c3, c4])
   =
   (
     (limbs 51 [a0, a1, a2, a3, a4])
     -
     (limbs 51 [b0, b1, b2, b3, b4])
   )
   (mod (2**255 - 19))
 &&
 and [
   c0 <u (2**51)@64,
   c1 <u (2**51)@64,
   c2 <u (2**51)@64,
   c3 <u (2**51)@64,
   c4 <u (2**51)@64
 ]
}
```

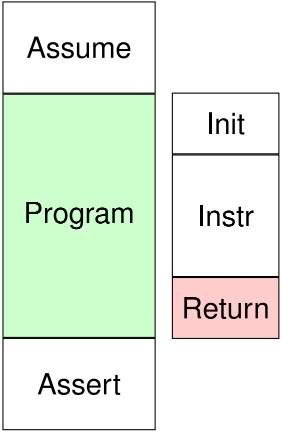
Assume
Program
Assert

$$\text{limbs}\,51\,[a_0, a_1, \cdots, a_n] = \sum_{i=0}^{n} a_i \times 2^{51\times i}$$

mod m: under modulo m

Verify by CRYPTOLINE - Return type casting - Revisit

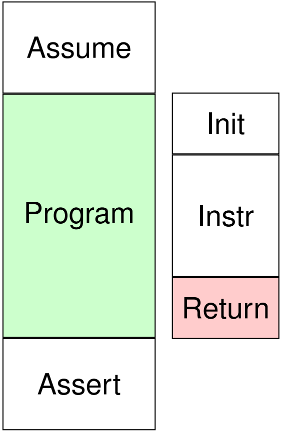
```
vpc c0@uint64 ioutput53_0@int64;
vpc c1@uint64 ioutput53_8@int64;
vpc c2@uint64 ioutput53_16@int64;
vpc c3@uint64 ioutput53_24@int64;
vpc c4@uint64 ioutput53_32@int64;
```



vpc: value preserve casting (will do safety check)

Verify by CRYPTOLINE - Return type casting - Revisit

```
vpc c0@uint64 ioutput53_0@int64;
vpc c1@uint64 ioutput53_8@int64;
vpc c2@uint64 ioutput53_16@int64;
vpc c3@uint64 ioutput53_24@int64;
vpc c4@uint64 ioutput53_32@int64;
```



vpc: value preserve casting (will do safety check)

Safety Check Failed

Counterexample generated by SMT solvers

```
(b4_0 (_ bv0 64))
(b3_0 (_ bv0 64))
(b2_0 (_ bv2251799813685250 64))
(b1_0 (_ bv0 64))
(b0_0 (_ bv2 64))
(a4_0 (_ bv0 64))
(a3_0 (_ bv1 64))
(a2_0 (_ bv0 64))
(a1_0 (_ bv1 64))
(a0_0 (_ bv2 64)) )
```

Figure: output by MathSAT

Counterexample generated by SMT solvers

```
(b4_0 (_ bv0 64))
(b3_0 (_ bv0 64))
(b2_0 (_ bv2251799813685250 64))
(b1_0 (_ bv0 64))
(b0_0 (_ bv2 64))
(a4_0 (_ bv0 64))
(a3_0 (_ bv1 64))
(a2_0 (_ bv0 64))
(a1_0 (_ bv1 64))
(a0_0 (_ bv2 64)) )
```

Figure: output by MathSAT

Found Counterexample translated to C language

```c
int main()
{
    felem in[5] = { 2, 1, 0, 1, 0 };
    felem out[5] = { 2, 0, 2251799813685250, 0, 0 };
    fdifference_backwards(out, in);
    for (int i = 0; i < 5; i++) {
        printf(" out%d: 0x%llx \n", i, out[i]);
    }
}
```

Check whether the program result is correct !

```
out0: 0x0
out1: 0x1
out2: 0x7ffffffffffe
out3: 0x7fffffffffff
out4: 0xffffffffffffffff
```

Check whether the program result is correct !

```
out0: 0x0
out1: 0x1
out2: 0x7fffffffffffe
out3: 0x7fffffffffffff
out4: 0xffffffffffffffff
```

- 0x0
- 0x1
- 0x5f6080e
- 0x0
- 0x0

Underflow and not in proper range

$$\texttt{0xffffffffffffffff} = 2^{64} - 1$$

Outline

Glimpse Evaluation Result

- 82 C functions (when paper is submitted)
- Evaluated on two different machines
  - much more range properties and safety check by SMT solver $\Rightarrow$ done in parallel
  - a few algebraic properties (most have only 1)
    - field operation
    - group operation

M1: Macbook 13”	2C/4T	16GB
M2: Ubuntu Server	18C/36T	1024GB

Evaluation Table - all functions

Table 2: Experimental Result

Function	L_{IR}	L_{CL}	D	P	TR_{M1}	MR_{M1}	TA_{M1}	MA_{M1}	TR_{M2}	MR_{M2}	TA_{M2}	MA_{M2}
nacl/curve25519/donna_c64/curve25519.c (MathSAT, SMT-LIB2 format)												
fdifference_backwards	69	69	66	0	-	-	0.23	6.3	-	-	0.14	9.1
fmul	91	127	10	14	12.51	452.2	0.20	6.3	4.05	486.6	0.14	9.3
fscalar_product	38	38	7	10	2.75	104.4	0.20	5.6	0.95	108.4	0.12	8.6
fsquare	68	116	10	12	7.44	288.1	0.22	6.3	2.61	301.0	0.13	9.3
fsum	20	20	0	0	0.48	5.6	0.15	4.8	0.22	10.0	0.10	8.2
fmonty	1147	1493	361	127	-	-	OOM	OOM	-	-	353.66	32764
wolfssl/fe_operations.c (Boolector with Lingeling, BTOR format)												
fe_add	40	40	0	0	1.48	6.5	0.19	5.6	0.61	9.5	0.11	8.6
fe_mul	305	305	20	24	OOT	OOT	0.32	7.0	13178	883.3	0.15	9.9
fe_mul121666	91	91	20	20	19.68	17.9	0.26	6.4	3.75	13.8	0.13	9.4
fe_neg	30	30	0	0	1.24	6.5	0.18	5.3	0.63	9.3	0.10	8.3
fe_sq	204	204	20	24	13411.84	351.9	0.33	6.7	2033	355.6	0.14	9.6
fe_sq2	214	214	20	24	18252.02	388.9	0.30	6.8	2763	385.5	0.14	9.6
fe_sub	40	40	0	0	1.31	6.5	0.16	5.7	0.64	9.4	0.11	8.6
curve25519[1]	2770	2770	200	236	OOT	OOT	12.06	385.6	68140	796.7	8.26	382.1
bitcoin/field_5x52_impl.h (MathSAT, SMT-LIB2 format)												
secp256k1_fe_add	13	20	0	0	0.33	5.3	0.14	4.8	0.22	10.0	0.09	8.3
secp256k1_fe_cmov	29	49	13	20	1.35	28.7	0.29	6.4	0.46	29.6	0.17	9.3
secp256k1_fe_from_storage	24	32	6	14	0.53	6.4	0.15	5.2	0.31	10.7	0.09	8.4
secp256k1_fe_mul_int	16	16	2	0	0.52	26.1	0.14	4.7	0.28	28.0	0.10	8.4
secp256k1_fe_negate	20	20	2	0	0.52	5.7	0.18	4.9	0.27	9.9	0.11	8.6
bitcoin/field_5x52_impl.h (Boolector with Lingeling, BTOR format)												
secp256k1_fe_normalize	52	60	21	0	117.18	45.3	0.12	5.3	91.89	31.5	0.08	8.3
secp256k1_fe_normalize_var	63	63	29	0	120.80	47.1	0.12	5.4	95.65	34.1	0.08	8.3
secp256k1_fe_normalize_weak	26	26	15	0	63.85	40.0	0.25	5.3	51.51	28.3	0.13	8.8
secp256k1_fe_normalizes_to_zero	34	39	10	0	203.12	60.3	0.16	5.2	151.03	42.9	0.08	8.2
bitcoin/field_5x52_int128_impl.h (MathSAT, SMT-LIB2 format)												
secp256k1_fe_mul_inner	111	137	17	24	16.09	461.0	0.22	6.5	4.00	489.1	0.14	9.5
secp256k1_fe_sqr_inner	90	116	21	22	9.91	284.5	0.20	6.4	2.72	303.2	0.14	9.3
bitcoin/scalar_4x64_impl.h (MathSAT, SMT-LIB2 format)												
secp256k1_scalar_add	81	102	55	22	2.03	10.1	0.21	6.5	1.11	14.1	0.13	9.4
secp256k1_scalar_eq	17	17	23	0	0.29	9.2	0.10	4.7	0.26	14.5	0.07	7.6
secp256k1_scalar_mul_512	273	384	136	90	13.75	263.3	0.26	7.1	4.96	280.0	0.16	9.9
secp256k1_scalar_mul	652	947	379	228	128.19	453.9	0.84	19.8	741.35	2219	0.43	16.3
secp256k1_scalar_negate	41	55	4	1	28.50	132.4	0.10	5.0	40.31	135.5	0.08	8.0
secp256k1_scalar_reduce_512	379	563	243	138	31.84	127.5	0.37	8.7	8.25	128.2	0.23	11.7
secp256k1_scalar_reduce	34	32	11	8	1.52	11.7	0.18	6.4	0.88	15.2	0.14	9.3
secp256k1_scalar_sqr_512	235	333	145	88	23.75	212.9	0.26	7.2	7.39	204.8	0.17	10.1
secp256k1_scalar_sqr	614	896	388	226	234.87	349.1	0.82	19.8	26.69	341.5	0.45	16.5
bitcoin/group_impl.h (MathSAT, SMT-LIB2 format)												
secp256k1_ge_from_storage	48	65	12	28	0.93	6.5	0.19	6.3	0.48	10.7	0.12	9.2
secp256k1_ge_neg	33	31	0	10	0.76	6.6	0.19	5.4	0.44	11.2	0.13	8.7
secp256k1_gej_add_ge_var	2109	2457	371	396	574.39	3166.9	OOM	OOM	75	3354	9363	70156
secp256k1_gej_double_var	899	1042	154	160	163.30	1703.0	0.77	18.4	25.27	1806	0.57	22.7
openssl/curve25519.c (MathSAT, SMT-LIB2 format)												
fe51_add	20	20	0	0	0.85	6.0	0.19	4.9	0.36	10.0	0.10	8.3
fe51_mul	96	105	11	20	17.95	381.2	0.26	6.4	3.69	409.3	0.13	9.2
fe51_mul121666	44	44	11	14	1.3	17.3	0.25	5.8	0.63	20.2	0.12	8.7
fe51_sq	73	82	11	20	8.07	227.0	0.23	6.3	2.22	247.6	0.14	9.2
fe51_sub	25	25	10	10	0.37	6.8	0.24	5.4	0.26	11.4	0.13	8.9
x25519_scalar_mult[1]	923	1047	110	194	558.56	1419.8	187.40	5538	119.89	1472	145.12	5511
openssl/ecp_nistp224.c (MathSAT, SMT-LIB2 format)												
felem_diff_128_64	24	36	0	0	0.56	6.4	0.23	5.1	0.32	10.7	0.14	8.6
felem_diff	24	24	0	0	0.55	5.8	0.19	4.9	0.33	10.4	0.11	8.8
felem_mul	40	40	0	0	2.24	83.2	0.15	5.2	0.65	88	0.09	8.2
felem_mul_reduce	82	121	15	16	10.65	321.8	0.20	6.4	3.11	322.5	0.13	9.1
felem_neg	47	58	5	10	0.95	6.8	0.19	5.8	0.55	11.1	0.12	8.7
felem_reduce	56	95	6	18	1.67	13.7	0.20	6.3	0.88	17.3	0.13	9.3
felem_scalar	12	12	0	0	0.48	26.7	0.14	4.6	0.24	28.9	0.09	8.1
felem_square	27	27	0	0	1.11	45.1	0.15	4.9	0.43	47.6	0.10	8.2
felem_square_reduce	69	108	14	18	6.36	195.8	0.21	6.4	1.81	198.8	0.13	9.2
felem_sum	16	16	0	0	0.41	5.4	0.15	4.7	0.26	10.0	0.10	8.3
widefelem_diff	41	63	0	0	0.90	6.5	0.19	5.7	0.46	10.6	0.12	8.7
widfefelem_scalar	21	21	0	0	2.58	87.7	0.14	4.8	0.70	88.3	0.10	8.4
openssl/ecp_nistp256.c (MathSAT, SMT-LIB2 format)												
felem_diff	24	36	0	0	0.59	7.6	0.18	5.1	0.35	11.7	0.12	8.6
felem_scalar	13	13	0	0	0.70	47.7	0.17	4.6	0.31	48.8	0.10	8.2
felem_shrink	65	95	18	16	1.78	14.0	0.20	6.4	0.95	17.1	0.13	9.3
felem_small_mul	145	95	17	46	4.75	123.0	0.23	7.0	2.29	123.2	0.14	9.8
felem_small_sum	20	20	0	0	0.41	5.8	0.14	4.8	0.25	10.2	0.10	8.4
felem_sum	16	16	0	0	0.41	5.6	0.14	4.7	0.24	10.3	0.09	8.2
smallfelem_mul	88	136	0	30	2.80	91.9	0.17	6.4	1.22	95.4	0.11	9.4
smallfelem_neg	26	28	0	0	0.1	5.4	0.19	4.9	0.27	9.7	0.12	8.6
smallfelem_square	60	108	0	20	1.92	55.8	0.15	6.3	0.85	55.5	0.10	9.2
openssl/ecp_nistp521.c[2] (MathSAT, SMT-LIB2 format)												
felem_diff64	45	45	18	18	0.81	6.9	0.20	6.4	0.48	11.4	0.13	9.3
felem_diff128	45	72	18	18	1.13	7.9	0.21	6.4	0.47	11.9	0.12	9.2
felem_neg	27	27	0	0	0.77	6.4	0.18	5.3	0.48	10.0	0.12	8.6
felem_reduce	122	155	74	72	4.10	7.8	0.24	6.7	2.06	10.8	0.14	9.6
felem_scalar	27	27	0	0	0.80	28.4	0.14	5.0	0.36	29.0	0.09	8.3
felem_scalar64	27	27	0	0	0.82	28.2	0.15	4.9	0.35	28.9	0.09	8.3
felem_scalar128	27	27	0	0	1.26	48.4	0.14	5.0	0.41	48.8	0.09	8.4
felem_sum64	36	36	0	0	0.49	6.0	0.14	5.2	0.29	10.0	0.10	8.3
felem_diff_128_64	54	54	0	0	1.34	7.2	0.29	6.0	0.68	11.4	0.15	8.7
felem_mul	188	188	0	0	23.92	187.0	0.22	6.6	3.13	182.5	0.13	9.5
felem_square	111	111	0	0	7.38	95.5	0.21	6.4	0.99	103.9	0.13	9.3
boringssl/fiat/curve25519.c (MathSAT, SMT-LIB2 format)												
fe_add	11	20	0	0	0.33	5.3	0.14	4.8	0.20	10.0	0.10	8.2
fe_mul_impl	96	108	9	22	18.39	452.9	0.21	6.4	5.11	473.9	0.13	9.2
fe_mul121666	43	43	9	14	1.12	18.4	0.20	5.7	0.62	21.2	0.11	8.6
fe_sqr_impl	73	85	9	22	10.59	278.7	0.26	6.3	3.11	293.0	0.12	9.2
fe_sub	15	25	0	0	0.51	5.9	0.19	5.0	0.28	10.4	0.11	8.8
x25519_scalar_mult_generic[1]	927	1073	161	212	470.68	1489.0	120.33	5726	118.95	1579	91.99	5766

Some comparisons

Montgomery Ladder step* involves 4 add, 4 sub, 4 square, 6 mul
(Curve25519) (field operations)

F	U/S	L_{IR}	L_{CL}	TR_{M1}	TA_{M1}	TR_{M2}	TA_{M2}	TH
openSSL	5 * U64	923	1047	9.3m	0.93s	2m	0.61s	
boringSSL	5 * U64	927	1073	7.8m	0.89s	2m	0.56s	
boringSSL	10 * U32	2715	3419	27.5m	59s	6.3m	42s	2h
wolfSSL	10 * S32	2770	2770	OOT	12s	18.9h	8s	

L_{IR}: lines of IR

L_{CL}: lines of CRYPTOLINE

TR(range, safety), TA(algebra): used time

OOT: used time $>$ 1day

TH: human effort (one person)

Montgomery Ladder is used for scalar multiplication of elliptic curve point

$$Q = aP$$

Outline

Conclusion

- A lightweight and easy to use method to verify cryptographic software involving both unsigned/signed operations.
- A GCC Plugin reducing human effort
- Verify several functions in well-known cryptographic libraries.
  - OpenSSL
  - BoringSSL
  - NaCl
  - wolfSSL
  - Bitcoin's libsecp256k1

CryptoLine Verifier

GCC Plugin

This Slide[1]

github.com/fmlab-iis

Signed Cryptographic Program Verification with Typed CRYPTOLINE

Open Access

[1] twleo.com/slides/ccs19-slide.pdf